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RADIATIVE HEAT TRANSFER IN A HEAT GENERATING AND TURBULENTLY CONVECTING FLUID LAYER

F. B. CHEUNG, S. H. CHAN,* T. C. CHAWLA and D. H. CHO
Reactor Analysis & Safety Division, Argonne National Laboratory,
9700 South Cass Avenue, Argonne, IL 60439, U.S.A.

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Abstract—The coupled problem of radiative transport and turbulent natural convection in a volumetrically heated, horizontal gray fluid medium, bounded from above by a rigid, isothermal wall and below by a rigid, adiabatic wall, is investigated analytically. An approximate method based upon the boundary layer approach is employed to obtain the dependence of heat transfer at the upper wall on the principal parameters of the problem, which, for moderate Prandtl number, are the Rayleigh number, Ra, the optical thickness, κL, and the conduction–radiation coupling parameter, N. Also obtained in this study is the behavior of the thermal boundary layer at the upper wall. At large κL, the contribution of thermal radiation to heat transfer in the layer is found to be negligible for $N > 10$, moderate for $N \sim 1$, and overwhelming for $N < 0.1$. However, at small κL, thermal radiation is found to be important only for $N < 0.01$. While a higher level of turbulence results in a thinner boundary layer, a larger effect of radiation is found to result in a thicker one. Thus, in the presence of strong thermal radiation, a much larger value of Ra is required for the boundary layer approach to remain valid. Under severe radiation conditions, no boundary layer flow regime is found to exist even at very high Rayleigh numbers. Accordingly, the ranges of applicability of the present results are determined and the approximate method justified. In particular, the validity of the present analysis is tested in three limiting cases, i.e. those of $\kappa L \to \infty$, $N \to \infty$, and $Ra \to \infty$, and is further confirmed by comparison with the numerical solution.

NOMENCLATURE

C_p,	specific heat;
$Ei(\chi)$,	ith exponential integral;
g,	acceleration due to gravity;
L,	layer depth;
M,	temperature ratio, equation (21);
N,	conduction–radiation coupling parameter;
Nu,	Nusselt number;
Nu_0,	Nusselt number for the purely turbulent convection case;
P,	pressure;
Pr,	Prandtl number;
q_R,	radiation flux;
Ra,	Rayleigh number;
S,	volumetric heat generation rate;
t,	time;
$\bar{T}$,	mean temperature;
ΔT,	temperature drop across the layer;
u,	velocity;
w,	vertical component of velocity;
z,	vertical coordinate.

Greek symbols

β,	isobaric coefficient of thermal expansion;
δ,	boundary layer thickness;
ρ_0,	density;
α,	thermal diffusivity;
ν,	kinematic viscosity;
λ,	thermal conductivity;
σ,	Stefan–Boltzmann constant;
κ,	absorption coefficient;
θ,	fluctuating temperature.

Subscripts

0,	lower surface;
1,	upper surface;
∞,	optically thick;
δ,	boundary layer.

1. INTRODUCTION

TURBULENT thermal convection in a horizontal fluid layer heated internally at high Rayleigh numbers [1] has in recent years received considerable interest owing to its technological and geophysical applications. So far, most theoretical and experimental investigations have been concerned with turbulent convection in low-temperature heat-generating fluid media. However, in many engineering problems such as the cooling of nuclear reactor core-melts, the fluid media in question are at high temperatures, usually of several thousand degree K. In such cases, the effect of internal thermal radiation can be quite significant on heat transport in the fluid layer and therefore requires proper considerations.

Very few studies have been made of radiative heat transfer in a volumetrically heated fluid layer, in spite

* Present address: Department of Mechanical Engineering, The University of Wisconsin-Milwaukee, Milwaukee, Wisconsin 53201, U.S.A.

of its practical importance. Anderson [2] employed the Rosseland diffusion approximation and radiative slip boundary conditions to obtain the radiant heat transfer in a horizontal molten uranium-oxide layer. Using empirically determined constant values for the apparent convective thermal diffusivity he found that heat transfer in the layer was enhanced markedly by radiation at moderate Rayleigh numbers. However, at high Rayleigh numbers, radiant heat loss appeared to be relatively small compared to the convective heat loss. Later, Cho *et al.* [3] employed the eddy heat transport model of Cheung [4] to account for the local variation of eddy thermal diffusivity in the layer. When combined with the Rosseland diffusion approximation, they found the contribution of thermal radiation to heat transfer in the layer to remain substantial even at high Rayleigh numbers. The assumption of a constant eddy thermal diffusivity in the layer especially in the wall region by Anderson [2] was clearly shown to underestimate the internal radiation effect in highly turbulent flow regime. Most recently Chawla *et al.* [5] performed a numerical study of the same problem using again Cheung's model [4] to describe the eddy heat flux but with an exact integral formulation for the radiation flux. The governing equations were solved by the method of collocation with piecewise polynomials as approximating functions to obtain the local temperature and surface heat transfer. Over the range of Rayleigh numbers considered (10^{10}–10^{14}), the radiation mode was demonstrated to be as important in heat transfer in the uranium-oxide layer as the turbulent thermal convection mode.

One of the characteristic features of turbulent thermal convection is the existence of a thin thermal boundary layer in the wall region, beyond which the variation in temperature is negligible. This boundary layer phenomenon, and other observations, have led some investigators [6, 7] to seek similarities between heat source-driven and Rayleigh–Bénard convection. However, with the effect of internal radiation, the boundary layer-dominant aspect may be strongly modified. The long range interaction of radiation tends to increase considerably the thickness of the thermal boundary layer. For forced convection flow, the radiative interactions in boundary layers have been discussed by many authors, such as Cess [8], Viskanta [9], Sparrow and Cess [10], Sibulkin and Dennar [11] and Venkateshan and Prasad [12]. For thermal convection flow, radiation effect on the boundary layer behavior is not known. In this paper, an approximate method based upon the boundary layer phenomenology is developed to study combined radiation and turbulent convection in a horizontal, heat generating fluid layer. Closed form expressions are derived for the boundary layer thickness and surface heat flux as functions of Rayleigh number and other parameters characterizing the nature of thermal radiation. The conditions for which the boundary layer approximation ceases to apply are discussed. Comparison of some of the present results are made with the numerical solution obtained by Chawla *et al.* [5].

2. THE GOVERNING EQUATIONS

We are concerned here with the upward heat transfer in a horizontal heat-generating fluid layer at high temperatures such as of interest to postaccident heat removal studies [5, 13]. The layer is assumed to be infinite in the horizontal extent and confined between a rigid, isothermal upper wall and a rigid, adiabatic lower wall. Both turbulent thermal convection and internal thermal radiation are considered to be important to heat transfer in the layer. The volumetric energy sources are assumed to be spatially uniform and time invariant so that the heat transfer process is statistically steady and one-dimensional. Furthermore, the fluid is assumed to be a gray medium with constant properties except density variation in the buoyancy force. Using the Boussinesq approximation, the governing equations are:

$$(\partial/\partial t - \alpha\nabla^2)T = -(\mathbf{u}\cdot\nabla)T + S/\rho_0 C_p - (\rho_0 C_p)^{-1}\nabla\cdot q_R, \quad (1)$$

$$(\partial/\partial t - \nu\nabla^2)\mathbf{u} = -(\mathbf{u}\cdot\nabla)\mathbf{u} - \rho_0^{-1}\nabla P + \mathbf{k}g\beta T, \quad (2)$$

$$\nabla\cdot\mathbf{u} = 0, \quad (3)$$

where $\mathbf{u}$ is the velocity vector, T the temperature, P the pressure, ρ_0 the mean density, S the volumetric rate of heat generation, C_p the specific heat, α the thermal diffusivity, ν the kinematic viscosity, g the gravitational acceleration, β the isobaric coefficient of thermal expansion, $\mathbf{k}$ the unit vector in the vertical, and q_R the radiation flux. Decomposing the dependent variables into the mean and fluctuating parts, i.e.

$$T = \bar{T} + \theta, \quad \bar{T} = \bar{T}(z), \quad \bar{\theta} = \mathbf{u} = 0, \quad (4)$$

where z is the vertical coordinate measured upward from the lower surface and the bar denotes ensemble or horizontal average, the governing equations become

$$\alpha\,\mathrm{d}^2\bar{T}/\mathrm{d}z^2 = \mathrm{d}\overline{w\theta}/\mathrm{d}z - \Phi + (\rho_0 C_p)^{-1}\overline{\nabla\cdot q_R}, \quad (5)$$

$$(\partial/\partial t - \alpha\nabla^2)\theta = -w\,\mathrm{d}\bar{T}/\mathrm{d}z - [(\mathbf{u}\cdot\nabla)\theta - \overline{(\mathbf{u}\cdot\nabla)\theta}] - (\rho_0 C_p)^{-1}[\nabla\cdot q_R - \overline{\nabla\cdot q_R}], \quad (6)$$

$$(\partial/\partial t - \nu\nabla^2)\mathbf{u} = -(\mathbf{u}\cdot\nabla)\mathbf{u} - \rho_0^{-1}\nabla p + \mathbf{k}g\beta\theta, \quad (7)$$

where $p = P - \bar{P} - \rho_0\overline{w^2}$ is the fluctuating pressure, w the velocity component in the vertical direction, and $\Phi = S/\rho_0 C_p$. For a parallel-plate geometry, the divergence of radiation flux may be expressed by

$$\nabla\cdot q_R = 4\kappa\sigma T^4 - 2\kappa\sigma[T_0^4 E_2(\kappa z) + T_1^4 E_2(\kappa L - \kappa z)] - \frac{\kappa^2}{\pi}\int_0^L\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\sigma T^4(r') \times \frac{\exp(-\kappa|r - r'|)}{(r - r')^2}\mathrm{d}x'\,\mathrm{d}y'\,\mathrm{d}z', \quad (8)$$

where κ is the absorption coefficient, σ the Stefan–Boltzmann constant, L the layer depth, T_0, T_1 the lower and upper wall temperatures, respectively, r the location of the fluid element in question, r' the location of the surrounding elements, and E_i the ith exponential integral. Averaging we have

$$\overline{\nabla \cdot q_R} = 4\kappa\sigma\bar{T}^4 - 2\kappa\sigma[T_0^4 E_2(\kappa z) + T_1^4 E_2(\kappa L - \kappa z)] - 2\kappa^2\sigma \int_0^L \bar{T}^4(\mu) E_1(|\kappa z - \kappa\mu|)\mathrm{d}\mu, \quad (9)$$

and

$$\nabla \cdot q_R - \overline{\nabla \cdot q_R} = 16\kappa\sigma\bar{T}^3\theta - \frac{4\kappa^2}{\pi}\int_0^L \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \times \sigma\bar{T}^3\theta \frac{\exp(-\kappa|r - r'|)}{(r-r')^2}\mathrm{d}x'\,\mathrm{d}y'\,\mathrm{d}z', \quad (10)$$

where the higher order terms involving $\theta/\bar{T}$ have been neglected. This is equivalent to assume $\Delta T/\bar{T} \ll 1$, which is consistent with the Boussinesq approximation. Equations (5) and (6) now become, respectively,

$$\alpha\,\mathrm{d}^2\bar{T}/\mathrm{d}z^2 = \mathrm{d}\overline{w\theta}/\mathrm{d}z - \Phi + \frac{\kappa\sigma}{\rho_0 C_p} \times \left\{4\bar{T}^4 - 2[T_0^4 E_2(\kappa z) + T_1^4 E_2(\kappa L - \kappa z)] - 2\kappa \int_0^L \bar{T}^4(\mu) E_1(|\kappa z - \kappa\mu|)\mathrm{d}\mu\right\}, \quad (11)$$

and

$$(\partial/\partial t - \alpha\nabla^2)\theta = -w\,\mathrm{d}\bar{T}/\mathrm{d}z - [(\mathbf{u}\cdot\nabla)\theta - \overline{(\mathbf{u}\cdot\nabla)\theta}] - \frac{\kappa\sigma}{\rho_0 C_p} \times \left\{16\bar{T}^3\theta - \frac{4\kappa}{\pi}\int_0^L\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \bar{T}^3\theta \times \frac{\exp(-\kappa|r - r|)}{(r-r')^2}\mathrm{d}x'\,\mathrm{d}y'\,\mathrm{d}z'\right\}. \quad (12)$$

With radiative interactions in the turbulently convecting fluid, equations (7), (11), and (12) constitute a basic system of equations governing the motion of the layer.

3. ANALYSIS

The principal goal is to predict heat transfer coefficient at the upper surface as a function of various controlling parameters. Here the Nusselt number is defined by:

$$Nu = \Phi L^2/\alpha\Delta T, \quad (13)$$

where the upper surface heat flux, ΦL, is a known quantity in the present problem, the unknown being the induced surface-to-surface temperature difference ΔT. Dimensional considerations of the governing equations indicate that the Nusselt number is a function of the Prandtl number, $Pr = \nu/\alpha$, of the fluid, the Rayleigh number,

$$Ra = g\beta\Phi L^5/2\alpha^2\nu, \quad (14)$$

of the layer defined in terms of the volumetric rate of heating, the optical thickness, κL, the surface temperature ratio T_0/T_1, and the conduction–radiation coupling parameter,

$$N = \lambda\kappa/4\sigma T_1^3, \quad (15)$$

where λ is the molecular thermal conductivity of the layer. In this study, we shall restrict ourselves to situations in which (i) the Rayleigh number is sufficiently high for turbulent flow to prevail, (ii) the Prandtl number is moderate* for the existence of a single (thermal and hydrodynamic) molecular boundary layer at the wall [1, 14], and (iii) the temperature ratio T_0/T_1 is of order unity for $\Delta T/\bar{T} \ll 1$. The corresponding lower limit of Ra shall be determined for different radiation conditions. Equation (11) may be integrated once from the bottom surface to an arbitrary location z using the boundary conditions

$$\overline{w\theta}(0) = \mathrm{d}\bar{T}/\mathrm{d}z|_{z=0} = 0, \quad (16)$$

to yield

$$\Phi z = -\alpha\,\mathrm{d}\bar{T}/\mathrm{d}z + \overline{w\theta} + \frac{4\kappa\sigma}{\rho_0 C_p}\left\{\int_0^z \bar{T}^4(\mu)E_2(\kappa z - \kappa\mu)\mathrm{d}\mu - \frac{1}{2}\int_0^L \bar{T}^4(\mu)[E_2(|\kappa\mu - \kappa z|) - E_2(\kappa\mu)]\mathrm{d}\mu - \frac{T_0^4}{2\kappa}\left[\frac{1}{2} - E_3(\kappa z)\right] - \frac{T_1^4}{2\kappa}[E_3(\kappa L - \kappa z) - E_3(\kappa L)]\right\}, \quad (17)$$

where the last term on the RHS of equation (17) is equivalent to the difference in mean radiation flux, $\bar{q}_R(z) - \bar{q}_R(0)$. Clearly, the local heat flux, Φz, is the sum of the conductive, convective, and radiative components.

a. *Boundary layer approximation*

In the turbulent flow regime, it has been observed that insofar as the radiation effects are not overwhelming, most changes of temperature in the layer occur in a relatively thin thermal boundary layer region at the upper wall. Over the body of the layer, the temperature is almost constant [3–5]. Accordingly, a simplified temperature profile may be assumed to evaluate the radiation component† in equation (17). This is

* Note that for extreme values of Pr, the thermal and the hydrodynamic boundary layers at the upper surface are likely to have different thickness, indicating the possibility of a rather weak correlation between w and θ [1]. Fortunately, the value of Pr for molten UO_2 is about 0.86, so that the present results may be directly applied to the studies of core-melt heat transfer [13].

† Since we have restricted ourselves to the case in which the boundary layer is very thin compared to the layer *as a whole*, the total error involved in this approach is very small, i.e. of order δ/L. This argument will be further examined based upon the analytical results achieved.

$$\bar{T}(z) = \begin{cases} T_0 & \text{for } 0 \le z < L - \delta \\ T_1\left[1 + (\Delta T/T_1)\left(\dfrac{L-z}{\delta}\right)\right] & \\ \text{for } L - \delta < z \le L, \end{cases} \tag{18}$$

where δ, the thermal boundary layer thickness, is an unknown and must be determined in the course of the analysis. In writing equation (18), we have assumed $\delta/L \ll 1$, which, as will be demonstrated later, is a valid approximation at sufficiently high Rayleigh number. By raising the above expression for $\bar{T}$ to the fourth power and simplifying according to the fact that for $(L - z) \sim \delta$,

$$(\Delta T/T_1)\left(\frac{L-z}{\delta}\right) \ll 1, \tag{19}$$

we obtain

$$\bar{T}^4(z) = \begin{cases} T_0^4 & \text{for } 0 \le z < L - \delta \\ T_1^4\left[1 + (4M\Delta T/T_1)\left(\dfrac{L-z}{\delta}\right)\right] & \\ \text{for } L - \delta < z \le L, \end{cases} \tag{20}$$

where M is a modified surface temperature ratio given by

$$M = \frac{1}{4}[1 + (T_0/T_1) + (T_0/T_1)^2 + (T_0/T_1)^3]. \tag{21}$$

Equation (20), which involves an error of the order $(\Delta T/T_1)^2$, satisfies the temperature continuity condition at $z = L - \delta$.

We may now investigate heat transport in the thermal boundary layer. Here, conduction and radiation are considered to dominate, so that the term $\overline{w\theta}$ in equation (17) may be neglected as the first approximation. Incorporating equation (20) into the radiation terms in equation (17) and carrying out the integrations, we obtain, after some manipulation,*

$$\begin{aligned} \Phi z = & -\alpha\, d\bar{T}/dz + (2M/N)(\alpha\Delta T/\delta) \\ & \times \{2/3 - E_4(\kappa L - \kappa z) - E_4[\kappa z - \kappa(L-\delta)] \\ & - E_4[\kappa(L-\delta)] + E_4(\kappa L)\}. \end{aligned} \tag{22}$$

Integration of the above equation once again over the boundary layer region using the approximate conditions, $\bar{T}(L) = T_1$ and $\bar{T}(L - \delta) = T_0$, yields a relation between the Nusselt number and the boundary layer thickness:

$$\begin{aligned} Nu = (\delta/L)^{-1}\Big\{1 + \frac{2M}{N}\Big[2/3 - \frac{2}{\kappa\delta}(1/4 - E_5(\kappa\delta)) \\ - E_4(\kappa(L-\delta)) + E_4(\kappa L)\Big]\Big\}, \end{aligned} \tag{23}$$

where the high order terms involving δ/L have been neglected. Obviously, an independent expression for the unknown boundary layer thickness is needed to determine the value of Nu. This is to be obtained by considerations of the fluctuating turbulence equations, (7) and (12).

For $L - \delta < z < L$, we may assume a balance between the viscous force and buoyancy in equation (7). This gives

$$|\nu\nabla^2\mathbf{u}| \sim |g\beta\theta| \quad \text{or} \quad \nu\langle w\rangle_\delta/\delta^2 \sim g\beta\langle\theta\rangle_\delta, \tag{24}$$

where $\langle\,\rangle_\delta$ denotes the rms value of the turbulence quantities in the δ region. In writing equation (24), we have assumed that the viscous boundary layer thickness is of the same order of the thermal boundary layer δ, which is probably true for moderate Prandtl number. In addition, we may assume that the conduction and radiation terms are of the same order as the production term in equation (12). This leads to

$$\begin{aligned} |\alpha\nabla^2\theta| + C_1Q_R \sim |w\, d\bar{T}/dz| \\ \text{or } \alpha\langle\theta\rangle_\delta/\delta^2 + C_1Q_R \sim \langle w\rangle_\delta\Delta T/\delta, \end{aligned} \tag{25}$$

where C_1 is a proportionality constant and Q_R is the magnitude of the radiation term given by

$$\begin{aligned} Q_R = \frac{\kappa\sigma}{\rho_0 C_p}\Big|16\bar{T}^3\theta - \frac{4\kappa}{\pi}\int_0^L\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\bar{T}^3\theta \\ \times\frac{\exp(-\kappa|r-r'|)}{(r-r')^2}dx'\,dy'\,dz'\Big|. \end{aligned} \tag{26}$$

The above expression may be simplified by assuming an average value for $\bar{T}^3\theta$ and performing the integration. This gives

$$\begin{aligned} &\int_0^L\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\bar{T}^3\theta\frac{\exp(-\kappa|r-r'|)}{(r-r')^2}dx'\,dy'\,dz' \\ &\simeq \langle\bar{T}^3\theta\rangle\int_0^L\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{\exp(-\kappa|r-r'|)}{(r-r')^2}dx'\,dy'\,dz' \\ &= 2\pi\langle\bar{T}^3\theta\rangle\int_0^L E_1(|\kappa z-\kappa z'|)dz' \\ &= 2\pi\langle\bar{T}^3\theta\rangle\kappa^{-1}[2 - E_2(\kappa z) - E_2(\kappa L-\kappa z)]. \end{aligned} \tag{27}$$

Equation (26) may thus be approximated by

$$Q_R \simeq \frac{\kappa\sigma}{\rho_0 C_p}|8\langle\bar{T}^3\theta\rangle[E_2(\kappa z) + E_2(\kappa L-\kappa z)]|, \tag{28}$$

where the terms involving $16\langle\bar{T}^3\theta\rangle$ have been properly cancelled. Mathematically, Q_R is the magnitude of the divergence of a gradient, as can be seen from equation (25). To estimate the value of Q_R in the δ region, we may simply integrate the RHS of equation (28) twice and average over the entire boundary layer, i.e.

$$\begin{aligned} Q_R = \frac{1}{\delta^2}\frac{8\kappa\sigma}{\rho_0 C_p}\iint_{L-\delta}^{L}\langle T^3\theta\rangle[E_2(\kappa z') \\ + E_2(\kappa L - \kappa z')]dz'\,dz. \end{aligned} \tag{29}$$

* Clearly, with radiation effects in the layer the temperature profile is highly nonlinear in the δ region, as indicated by equation (22). However, equation (20) can still be a reasonable approximation provided that the condition $\delta/L \ll 1$ is met under various radiation conditions.

Using an average value for $\langle \bar{T}^3\theta \rangle$, we obtain

$$Q_R = \frac{8\kappa\sigma}{\rho_0 C_p} \frac{1}{(\kappa\sigma)^2} \{1/3 - E_4(\kappa\delta) + E_4(\kappa L) - E_4[\kappa(L-\delta)]\} \bar{T}^3_{av} \langle \theta \rangle_\delta. \quad (30)$$

In view of equation (20), we have $\bar{T}^3_{av} = M T_1^3$. Equation (25) becomes

$$\alpha\langle\theta\rangle_\delta/\delta^2 + C_1 \frac{8\kappa\sigma}{\rho_0 C_p} \frac{M T_1^3}{(\kappa\delta)^2} \{1/3 - E_4(\kappa\delta) + E_4(\kappa L) - E_4[\kappa(L-\delta)]\}\langle\theta\rangle_\delta \sim \langle w \rangle_\delta \Delta T/\delta, \quad (31)$$

where an additional constant may be absorbed in C_1 to ensure a correct choice of $\bar{T}_{av}$. Equations (24) and (31) lead to

$$(\delta/L)^{-1} = C_2^{4/3} Ra^{1/3}\{1 + (2C_1 M/N) \times [1/3 - E_4(\kappa\delta) + E_4(\kappa L) - E_4(\kappa(L-\delta))]\}^{-1/3} Nu^{-1/3}, \quad (32)$$

where C_2 is a proportionality constant* and the 4/3 power is chosen for convenience. The Nusselt number may now be determined by combination of equations (23) and (32). This gives

$$Nu = C_2 Ra^{1/4}\{1 + (2C_1 M/N)[1/3 - E_4(\chi) + E_4(\kappa L) - E_4(\kappa L - \chi)]\}^{-1/4} \times \{1 + (2M/N)[2/3 - 2\chi^{-1}(1/4 - E_5(\chi)) - E_4(\kappa L - \chi) + E_4(\kappa L)]\}^{3/4}, \quad (33)$$

where $\chi = \kappa\delta$ is given implicitly by

$$\chi = C_2^{-1} Ra^{-1/4}(\kappa L)\{1 + (2C_1 M/N) [1/3 - E_4(\chi) + E_4(\kappa L) - E_4(\kappa L - \chi)]\}^{1/4} \times \{1 + (2M/N)[2/3 - 2\chi^{-1}(1/4 - E_5(\chi)) - E_4(\kappa L - \chi) + E_4(\kappa L)]\}^{1/4}. \quad (34)$$

Thus, Nu is a function of Ra, κL, M and N. Once χ is known, the thermal boundary layer thickness can be calculated by the simple relation $\delta/L = (\kappa L)^{-1}\chi$. Before proceeding to numerical solution, however, the two unknown constants C_1 and C_2 have to be determined. Also, the validity of equation (30) and thus (32) requires further justification to substantiate the foregoing physical and mathematical arguments.

b. *Comparison of some special cases*

If we set $\kappa \to \infty$ in equation (33), we may obtain an expression for the Nusselt number in the optically thick limit,† i.e.

$$Nu_\infty = C_2 Ra^{1/4}(1 + 2C_1 M/3N)^{-1/4} \times (1 + 4M/3N)^{3/4}. \quad (35)$$

However, for an optically thick medium, a simple differential approximation may be employed for the radiation flux [10, 15]. This gives

$$\overline{\nabla \cdot q_R} = -\frac{4\sigma}{3\kappa}\frac{d^2\bar{T}^4}{dz^2}, \quad (36)$$

and

$$\nabla \cdot q_R - \overline{\nabla \cdot q_R} = -\frac{4\sigma}{3\kappa}\nabla^2(4\bar{T}^3\theta), \quad (37)$$

where $\theta/\bar{T} \ll 1$ has been assumed. Equations (11) and (12) now become

$$\alpha d^2/dz^2[(1 + 4\sigma\bar{T}^3/3\lambda\kappa)\bar{T}] = d\overline{w\theta}/dz - \Phi, \quad (38)$$

and

$$\partial\theta/\partial t - \alpha\nabla^2[(1 + 16\sigma\bar{T}^3/3\lambda\kappa)\theta] = -w\, d\bar{T}/dz - [(\mathbf{u}\cdot\nabla)\theta - \overline{(\mathbf{u}\cdot\nabla)\theta}], \quad (39)$$

respectively. Performing the boundary layer analysis as done before, equation (38) leads to

$$Nu_\infty = (\delta/L)^{-1}(1 + 4M/3N), \quad (40)$$

whereas equation (39) leads to

$$(\delta/L)^{-1} = C_2^{4/3} Ra^{1/3}(1 + 4M/3N)^{-1/3} Nu_\infty^{-1/3}. \quad (41)$$

Thus we have

$$Nu_\infty = C_2 Ra^{1/4}(1 + 4M/3N)^{1/2}, \quad (42)$$

where the unknown coefficient is purposely chosen to be C_2 for convenience. Comparison of equations (35) and (42) suggests $C_1 = 2$. Note that the simplified temperature profile of equation (20) has not been used in the derivation of equations (40) and (41). Here the factor $(1 + 4M/3N)$ comes directly from integration of the LHS of equation (38), i.e.

$$\int_{T_1}^{T_0} d[(1 + 4\sigma\bar{T}^3/3\lambda\kappa)\bar{T}] = \Delta T + (4\sigma/3\lambda\kappa)(T_0^4 - T_1^4) = \Delta T(1 + 4M/3N), \quad (43)$$

where M is given by equation (21). The proper behavior of equation (33) in the optically thick limit clearly supports the various arguments employed in the approximate analysis. In particular, this indicates that equation (30) is an adequate expression for Q_R in equation (25). From equation (39), the ratio of the magnitudes of the radiation and conduction terms is obviously equal to

$$R = 16\sigma\bar{T}^3_{av}/3\lambda\kappa, \quad (44)$$

where the average temperature, $\bar{T}_{av}$, has a value between T_0 and T_1. If we set $\kappa \to \infty$ in equation (30) and take $C_1 = 2$ in equation (25), we have

*This is true only for the case of moderate Prandtl numbers, i.e. $Pr \sim 1$. For very large or very small values of Pr, C_2 may be a strong function of Pr. In such cases, care must be taken in employing the present results. Whether the condition, $\delta/L \ll 1$, is valid or not depends not only upon the Rayleigh number, but also upon the Prandtl number.

† We shall consider the limit in which the boundary layer *itself* is optically thick, i.e. $\chi \to \infty$. This condition is met if $\kappa L \gg (\delta/L)^{-1}$, as will be further discussed in the subsequent sections.

$$\frac{C_1 Q_R}{\alpha\langle\theta\rangle_\delta/\delta^2} = \frac{2(8\kappa\sigma/\rho_0 C_p)(\kappa\delta)^{-2}(\bar{T}_{av}^3\langle\theta\rangle_\delta/3)}{\alpha\langle\theta\rangle_\delta/\delta^2} = R, \tag{45}$$

which demonstrates the self-consistency of the analysis.

Another asymptotic case may be obtained if we set $N \rightarrow \infty$ in equation (33), corresponding to the case of negligible radiation effect. This gives

$$Nu_0 = C_2\, Ra^{1/4}, \tag{46}$$

which in essence is the asymptotic law of heat transport in the absence of internal radiation. The above relation is valid only for very high Rayleigh number flows. At lower Rayleigh numbers, a modified relation has been obtained by Cheung [1]:

$$Nu_0 = \frac{0.206\, Ra^{1/4}}{1 - 0.847\, Ra^{-1/12}}. \tag{47}$$

However, as $Ra \rightarrow \infty$, we have $Nu_0 \rightarrow 0.206\, Ra^{1/4}$. This suggests a choice of $C_2 = 0.206$. Nevertheless, significant deviation from equation (46) may arise if Ra is not sufficiently high, as implied by equation (47). Since the radiation effect tends to thicken the thermal boundary layer, the use of equation (33) at lower Rayleigh numbers may result in even more severe error. Obviously, under certain conditions, the convective component, $\overline{w\theta}$, may not be too small to be neglected in the δ region, especially when strong radiative interaction is present.

c. *Correction for higher order effect*

To account for the secondary enhancement in heat transfer by the relatively weak thermal mixing in the δ region, the eddy heat transport model of Cheung [4] is employed to improve upon the expression of equation (22). From [4], we have

$$\frac{\overline{w\theta}}{-\alpha\, d\bar{T}/dz} = 0.051\left[\frac{g\beta(\bar{T}-T_1)L^3}{\alpha\nu}\right]^{0.87}\left[\frac{z}{L}\left(1-\frac{z}{L}\right)\right]^3 \tag{48}$$

Using the simplified temperature profile of equation (18), we have, for $L - \delta \le z < L$,

$$\overline{w\theta} = C_3(\alpha\Delta T/\delta)[0.051(g\beta\Delta T L^3/\alpha\nu)^{0.87} \times (\delta/L)^{-0.87}(1 - z/L)^{3.87}], \tag{49}$$

where C_3 is a proportionality constant and where we have assumed $(z/L)^3 = 1 - O(\delta/L) \simeq 1$. Equation (22) now becomes

$$\begin{aligned}\Phi z = &-\alpha\, d\bar{T}/dz + C_3(\alpha\Delta T/\delta)][0.051\\ &\times (g\beta\Delta T L^3/\alpha\nu)^{0.87}(\delta/L)^{-0.87}(1-z/L)^{3.87}]\\ &+ (2M/N)(\alpha\Delta T/\delta)\{2/3 - E_4(\kappa L - \kappa z)\\ &- E_4[\kappa z - \kappa(L-\delta)]\\ &- E_4[\kappa(L-\delta)] + E_4(\kappa L)\},\end{aligned} \tag{50}$$

which, upon integration, yields

$$\begin{aligned}Nu = &(\delta/L)^{-1}\{1 + 0.019 C_3(Ra/Nu)^{0.87}\\ &\times (\delta/L)^3 + (2M/N)[2/3 - (2/\kappa\delta)(1/4 - E_5(\kappa\delta))\\ &- E_4(\kappa(L-\delta)) + E_4(\kappa L)]\}.\end{aligned} \tag{51}$$

In obtaining the above equation, the higher order terms involving δ/L have been neglected. Equations (32) and (51), when combined, lead to an implicit expression for the Nusselt number:

$$\begin{aligned}Nu = &C_2 Ra^{1/4}\{1 + (2C_1 M/N)[1/3 - E_4(\chi) + E_4(\kappa L)\\ &- E_4(\kappa L - \chi)]\}^{-1/4}\{1 + 0.019(C_3/C_2^4)\\ &\times\{1 + (2C_1 M/N)[1/3 - E_4(\chi) + E_4(\kappa L)\\ &- E_4(\kappa L - \chi)]\}(Ra/Nu)^{-0.13} + (2M/N)\\ &\times[2/3 - 2\chi^{-1}[1/4 - E_5(\chi)]\\ &- E_4(\kappa L - \chi) + E_4(\kappa L)]\}^{3/4},\end{aligned} \tag{52}$$

where $\chi = \kappa\delta$ is again given implicitly by

$$\begin{aligned}\chi = &C_2^{-1} Ra^{-1/4}(\kappa L)\{1 + (2C_1 M/N)[1/3 - E_4(\chi)\\ &+ E_4(\kappa L) - E_4(\kappa L - \chi)]\}^{1/4}(1 + 0.019(C_3/C_2^4)\\ &\times\{1 + (2C_1 M/N)[1/3 - E_4(\chi) + E_4(\kappa L)\\ &- E_4(\kappa L - \chi)]\}(Ra/Nu)^{-0.13} + (2M/N)\\ &\times\{2/3 - 2\chi^{-1}[1/4 - E_5(\chi)]\\ &- E_4(\kappa L - \chi) + E_4(\kappa L)\})^{1/4}.\end{aligned} \tag{53}$$

As before, the constants C_1 and C_2 can be determined by considerations of the limiting behavior of Nu. Comparison with the case of $\kappa L \rightarrow \infty$, we obtain $C_1 = 2$, whereas with the case of $N \rightarrow \infty$ and $Ra \rightarrow \infty$, we have $C_2 = 0.206$. To determine C_3, we may consider the case in which N approaches infinity but not Ra. From equation (52), we get, for the purely turbulent thermal convection case,

$$Nu_0 = 0.206\, Ra^{1/4}[1 + 10.61\, C_3 \times (Ra/Nu_0)^{-0.13}]^{3/4}. \tag{54}$$

Using the water data ($3 \lesssim Pr \lesssim 6$) of Kulacki and Emara [16], the constant is found to be $C_3 = 0.210$. A plot of $Ra^{1/4}/Nu_0$ vs. Ra is shown in Fig. 1, with the solid line representing the theoretical curve of equa-

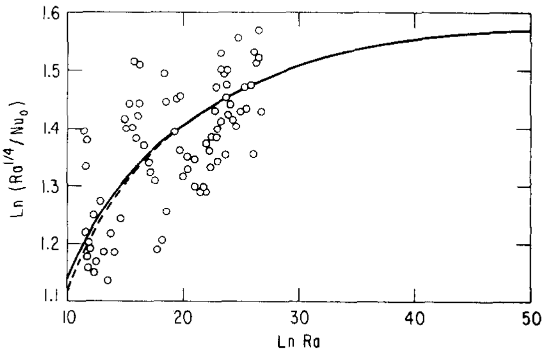

FIG. 1. $Ra^{1/4}/Nu_0$ vs Ra for $N = \infty$. ——— theoretical curve of equation (54); – – – – prediction by Cheung [1]; ○ data of Kulacki and Emara [16].

tion (54); the dashed line representing that of equation (47), and the data points from the heat transfer measurements of Kulacki and Emara. In view of the wide range of Ra covered, the various sets of results are considered to agree satisfactorily among themselves. Note that the scatter of the data is artifically amplified in the $Ra^{1/4}/Nu_0$–Ra plot. If presented in the conventional Nu_0–Ra space, the deviation of the data from the theoretical curves would only be of few percent over the entire Ra range, and the correlation would appear to be almost perfect. Also, the small difference between the two theoretical curves as appears in the lower left hand corner of the figure would not be even detected if plotted in the conventional Nu_0–Ra scale. For large values of Ra, equations (47) and (54), which are semi-empirically based, i.e. rely on existing measurements and the boundary layer assumptions, may be approximated by

$$Nu_0 = 0.206\, Ra^{1/4}(1 + 0.847\, Ra^{-1/12}), \quad (55)$$

and

$$Nu_0 = 0.206\, Ra^{1/4}[1 + 1.671(Ra/Nu_0)^{-0.13}], \quad (56)$$

respectively, by simply expanding the original expressions asymptotically. As can be seen from Fig. 1, the two expressions are practically the same although they were derived from two entirely different approaches (that of Cheung [1] and the one presently employed). Comparison of equations (46) and (56) indicates that at lower Rayleigh numbers thermal mixing by $\overline{w\theta}$ in the δ region may be responsible for the deviation of Nu_0 from the asymptotic behavior.

Equations (33) and (34), which may be obtained directly from equations (52) and (53) by neglecting those terms involving $(Ra/Nu)^{-0.13}$, represent the asymptotic law of heat transport in the presence of internal thermal radiation. For a given set of Ra, κL, M and N, these equations may be solved numerically by a standard nonlinear subroutine solver available in any computing laboratory. The value of Nu so obtained may then be used as an initial trial value for the solution of equations (52) and (53).

4. RESULTS AND DISCUSSION

Although the present analysis has resulted in four independent controlling parameters, only three of them need to be considered. These are the Rayleigh number, Ra, the optical thickness, κL, and the conduction–radiation coupling parameter, N. As can be seen in equations (52) and (53), the parameter M always appears with N as a group, i.e. $2M/N$. Since we have required $\Delta T/\bar{T}$ to be small in this study, the variation of M is limited to values close to unity. Consequently, the effect of M may easily be absorbed in N. Throughout the present calculation, a constant value of $M = 1.34$, corresponding to a surface temperature ratio of $T_0/T_1 = 1.2$, is assumed. Physically, this represents the maximum temperature ratio which is possible to attain in a molten uranium-oxide [20].

In Fig. 2, a plot of Nu vs. Ra is presented for $N = 0.4$, $\kappa L = 1000$ and $1 \times 10^8 \le Ra \le 1 \times 10^{13}$. These values of parameters are chosen since they are typical of those encountered in many practical applications such as nuclear reactor safety studies. The prediction of equation (52) is given by the solid line whereas that of equation (33) by the dashed line. At lower Rayleigh numbers, there is indeed considerable difference between the two predicted behaviour of Nu. However, as Ra increases, the difference becomes smaller, and eventually vanishes for $Ra > 1 \times 10^{16}$ (not shown in the figure), indicating a negligible effect of $\overline{w\theta}$ inside the thermal boundary layer beyond this Rayleigh number. The numerical results of Chawla *et al.* [5] are given by the dotted circles. While in the present analysis, the eddy heat transport model of Cheung [4] was employed only in the δ region to account for the secondary convective effect, it was employed in the study of Chawla *et al.* to determine the temperature field and thus the heat transfer in the layer as a whole. The excellent agreement between the two sets of predicted results as shown in the figure strongly recommends that both approaches are physically sound and valid. In the absence of thermal radiation, the purely turbulent thermal convection case which is also shown in Fig. 2 represents the lower limit of Nu.

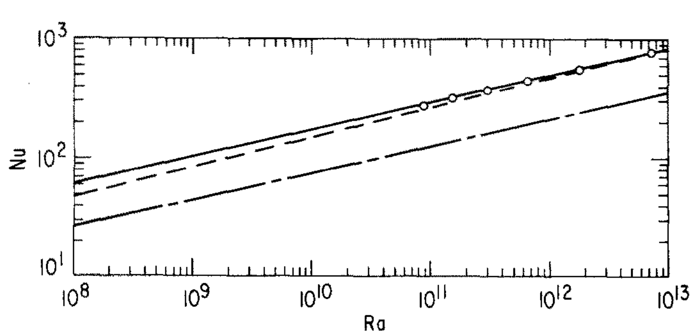

FIG. 2. Nu vs Ra for $N = 0.4$ and $\kappa L = 1000$. ——— theoretical curve of equation (52); – – – theoretical curve of equation (33); —-— purely convection case; ○ numerical data of Chawla *et al.* [5].

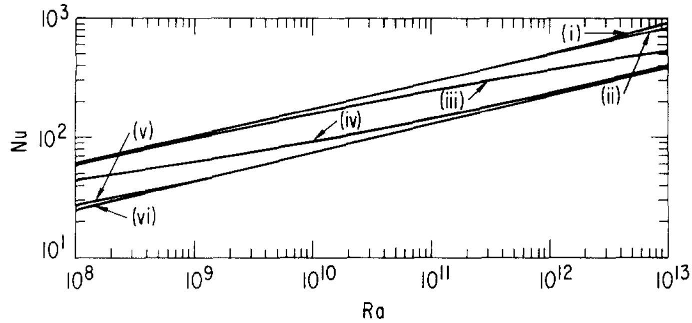

FIG. 3. Nu vs Ra for $N = 0.4$: (i) $\kappa L = 10^4$; (ii) $\kappa L = 10^3$; (iii) $\kappa L = 10^2$; (iv) $\kappa L = 10$; (v) $\kappa L = 1$; (vi) $\kappa L = 0.1$.

Contrary to the finding of Anderson [2] that concludes a diminishing effect of radiation at high Rayleigh numbers, here the radiation effect on heat transfer is found to be as important at high Ra as that at low Ra.

Figures 3 and 4 display the effects of optical thickness and conduction–radiation coupling parameter on the variation of Nu with Ra, respectively. For a given value of $N = 0.4$, the rate of heat transfer is found to increase with the optical thickness in the range of $1 < \kappa L < 1000$, as can be seen in Fig. 3. However, for $\kappa L < 1$ and $\kappa L > 1000$, Nu is found to be almost independent of κL except at rather low (i.e. 1×10^8) or high Ra (i.e. 1×10^{13}). To explain this, we have to further examine equation (52). Since we have $\delta/L \ll 1$ in the turbulent regime, the following can be a good approximation:

$$E_4(\kappa L) - E_4(\kappa L - \chi) = E_4(\kappa L) - E_4[\kappa L(1 - \delta/L)] = O(\delta/L). \quad (57)$$

Equation (52) thus reduces to

$$Nu = 0.206 Ra^{1/4}\{1 + (4M/N)[1/3 - E_4(\chi)]\}^{-1/4}\{1 + 2.228[1 + (4M/N) \times [1/3 - E_4(\chi)](Ra/Nu)^{-0.13} + (2M/N)[2/3 - 2\chi^{-1}](1/4 - E_5(\chi))]\}^{3/4}, \quad (58)$$

which shows a dependence of Nu upon χ but not directly upon κL. Physically, it is the optical thickness of the thermal boundary layer that assumes the role of thermal radiation, not the optical thickness of the layer itself. This result is quite reasonable since most of the change in temperature occurs in the δ region. The effect of κL is merely to modify the value of χ, as given by equation (53). Now let us return to Fig. 3. As to be discussed later, the value of δ/L for $N = 0.4$ is of the order of 0.03. Hence for $\kappa L < 1$ or $\chi < 0.03$, the *boundary layer* is optically thin; and for $\kappa L > 1000$ or $\chi > 30$ it is optically thick (in practical sense). As a result, marked effect of κL can be seen only in the range of $1 < \kappa L < 1000$ corresponding to $0.03 < \chi < 30$. For $Ra \geq 10^{13}$, however, we have $\delta/L < 0.03$ or $\chi < 30$, so that the boundary layer is still not optically thick enough for the effect of κL to vanish even at $\kappa L \sim 1000$. Similar argument may be applied to the lower ends of Ra and κL. For $\kappa L = 100$, corresponding to $\chi \sim 1$, the effect of N on surface heat transfer is shown in Fig. 4. Relative to that of κL, the effect of N is found to

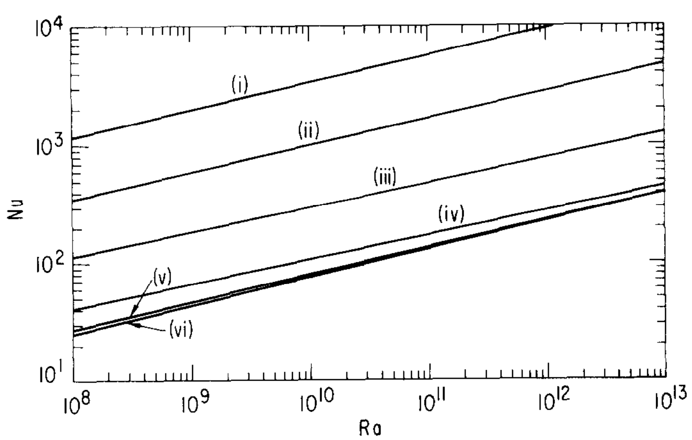

FIG. 4. Nu vs Ra for $\kappa L = 100$: (i) $N = 0.001$; (ii) $N = 0.01$; (iii) $N = 0.1$; (iv) $N = 1$; (v) $N = 10$; (vi) $N = 100$.

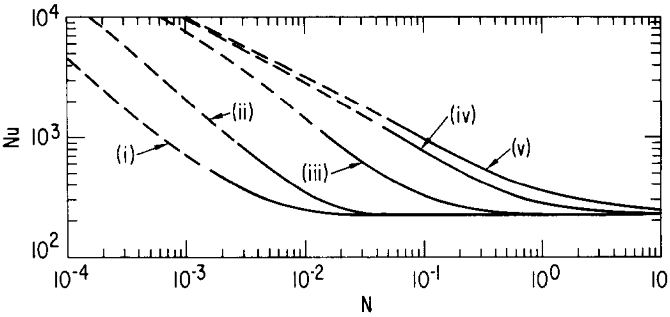

FIG. 5. Nu vs N for $Ra = 1 \times 10^{12}$: (i) $\kappa L = 0.1$; (ii) $\kappa L = 1$; (iii) $\kappa L = 10$; (iv) $\kappa L = 100$; (v) $\kappa L = 1000$.

be much larger. Thus for large values of κL, N may be considered as the principal radiation parameter of the problem [3]. For $N > 1$, the radiation effect is only moderate on Nu, and decreases slightly as Ra is increased. Beyond $N = 10$ or 100, the effect is practically zero. On the other hand, for $N < 1$, the effect is always important, regardless of the value of Ra. The radiation mode is in fact a dominating one for $N < 0.1$, where Nu is found to vary roughly as $N^{-1/2}$.

In Fig. 5, the variation of Nu with N is presented for different κL, while keeping the value of Ra constant at 1×10^{12}. At this Rayleigh number, the radiation effect is found to be negligible for $N > 10$ and remain so for $0.01 < N < 1$ if $\kappa L < 1$. However, as κL is increased and N decreased, the effect may become overwhelming. For $N > 1$, sensible variation of Nu is obtained over the range of relatively large κL, i.e. $10 < \kappa L < 1000$. The Nusselt number becomes sensitive to smaller κL only when N is small. For $N < 0.01$, Nu is found to vary markedly at $\kappa L \sim 1$ whereas it is almost independent of κL for $\kappa L > 100$. In general, the effect of κL on Nu increases with decreasing N, indicating the significance of both parameters on the rate of heat transfer. Note that for small N and large κL corresponding to cases with strong radiation effect, the thermal boundary layer may become too thick for the present results to remain valid. As shown in Fig. 6, the requirement of $\delta/L \ll 1$ is met only for reasonable values of N and κL. If we arbitrarily set the criterion as $\delta/L \leq 0.03$, the range of applicability of the present analysis would be limited to the lower region of the figure. In particular, for $\kappa L \geq 100$, we require $N \geq 0.04$. Beyond this range, the results obtained are physically doubtful, as represented by the dashed lines in Fig. 5. Thus under severe thermal radiation conditions, the boundary layer phenomenon in thermal convection may not exist even at very high Rayleigh numbers. This is further illustrated in Fig. 7, which shows the variation of δ/L with Ra for a number of selected values of κL and N. In this figure, the effect of thermal radiation is strongest in case (i), decreasing gradually with increasing case number, and is weakest in case (v). For a given value of Ra, the boundary layer is found to be thicker as radiation effect gets larger. Conversely, in all cases δ/L is found to decrease monotonically with increasing Ra. Thus the radiation and turbulent convection modes tend to counteract each other. For the boundary layer approximation to

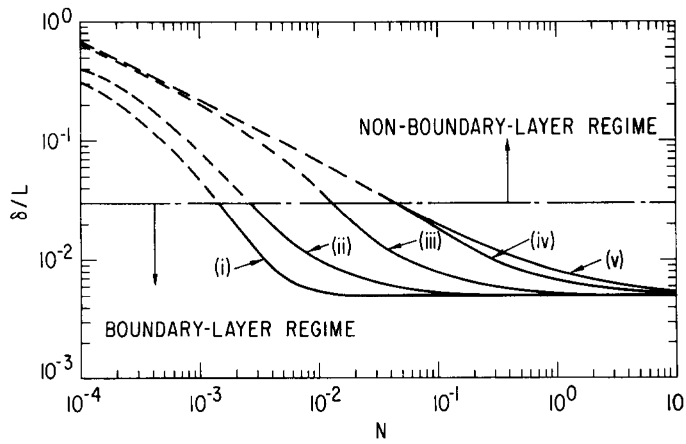

FIG. 6. δ/L vs N for $Ra = 1 \times 10^{12}$: (i) = 0.1; (ii) $\kappa L = 1$; (iii) $\kappa L = 10$; (iv) $\kappa L = 100$; (v) $\kappa L = 1000$.

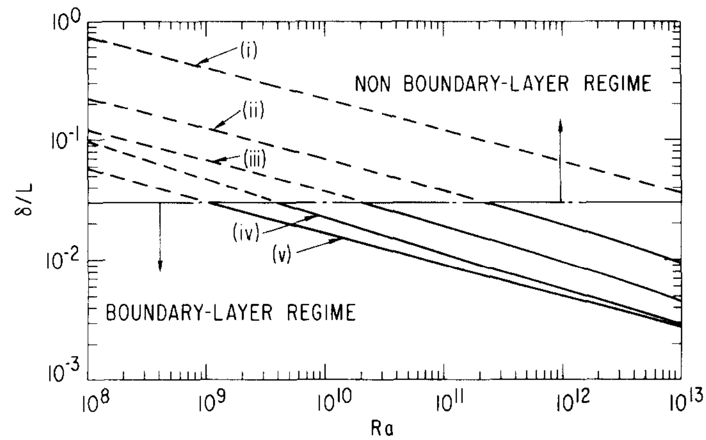

FIG. 7. δ/L vs Ra: (i) $N = 0.01, \kappa L = 100$; (ii) $N = 0.1, \kappa L = 100$; (iii) $N = 0.4, \kappa L = 100$; (iv) $N = 0.4, \kappa L = 10$; (v) $N = 0.4$, $\kappa L = 1$.

apply, the Rayleigh number is required to be sufficiently high so as to induce strong turbulent mixing in the layer to offset the radiative interaction. If this is not satisfied, as in case (i), the boundary layer regime ceases to exist. Fortunately, in many engineering problems such as postaccident heat removal in nuclear reactors, N is usually larger than 0.1 and Ra higher than 1×10^{11}, so that the criteria can easily be met.

5. CONCLUSION

Thermal radiation has been demonstrated to be a significant heat transfer mode in heat source-driven thermal convection at high temperatures. The radiation enhanced heat transfer is found to be as important at high Rayleigh numbers as that at low Rayleigh numbers. In general, large radiation effect is obtained at large optical thickness and small conduction–radiation coupling parameter. For moderate values of κL and N, heat transfer is found to be equally sensitive to changes in both parameters.. However, the Nusselt number is only indirectly dependent upon κL through its dependence on the optical thickness of the thermal boundary layer. Consequently, for $\kappa L < 1$ or $\kappa L > 1000$, corresponding in general to $\chi < 0.03$ or $\chi > 30$, respectively, the boundary layer becomes optically too thin or too thick for the effect of κL to remain substantial. Marked variation of Nu with κL is obtained only in the range $1 < \kappa L < 1000$. For large values of χ, corresponding to $\kappa L \geq 1000$, the effect of radiation is found to be negligible for $N > 10$, moderate for $N \sim 1$, and dominating for $N < 0.1$. On the other hand, for small values of χ, corresponding to $\kappa L \leq 1$, considerable radiation effect is obtained only if $N < 0.01$. Finally, it has been shown that the present analysis is a valid one provided that the Rayleigh number is high enough to sustain a thin thermal boundary layer at the upper wall. For the case in which the radiation effect is not overwhelming, i.e. $N \geq 0.1$, the lower limit of Ra for the validity of the boundary layer approximation is found to be roughly equal to 1×10^{11}, which fortunately is still below or among those values commonly encountered in many engineering applications.

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TRANSFERT THERMIQUE RADIATIF DANS UNE COUCHE DE FLUIDE TURBULENTE GENERATRICE DE CHALEUR

Résumé—On étude théoriquement le problème couplé du rayonnement et de la convection naturelle turbulente dans un milieu de fluide gris horizontal, chauffé dans sa masse, limité au dessus par une paroi rigide et isotherme, au dessous par une paroi rigide et adiabatique. Une méthode basée sur l'approche de la couche limite est utilisée pour obtenir l'influence, sur le transfert thermique à la paroi supérieure, des paramètres principaux du problème qui, pour des nombres de Prandtl modérés, sont le nombre de Rayleigh Ra, l'épaisseur optique κL et le paramètre de couplage entre conduction et rayonnement N. On obtient aussi le comportement de la couche limite thermique sur la paroi supérieure. Pour κL grand, la contribution du rayonnement thermique dans le transfert thermique est négligeable pour $N > 10$, modérée pour $N \sim 1$ et assez forte pour $N < 0{,}1$. Néanmoins, aux faibles valeurs de κL, le rayonnement thermique est important seulement si $N < 0{,}01$. Tandis qu'un niveau de turbulence élevé s'accompagne d'une couche limite plus mince, un effet important du rayonnement accompagne une couche plus épaisse. En présence d'un rayonnement thermique important, une plus grande valeur de Ra est nécessaire pour valider l'approche choisie. Dans des conditions sévères de rayonnement, il n'y a pas de régime de couche limite, même à de très grand nombres de Rayleigh. Les domaines d'application des résultats présentés sont déterminés et la méthode d'approche est justifiée. En particulier, la validité de l'analyse est testée dans trois cas extrêmes, ceuz pour lesquels $\kappa L \to \infty$, $N\ \infty$, et $Ra \to \infty$, en comparaison avec la solution numérique.

STRAHLUNGSWÄRMEAUSTAUSCH IN EINER TURBULENT STRÖMENDEN FLÜSSIGKEITSSCHICHT MIT INNEREN WÄRMEQUELLEN

Zusammenfassung—Das gekoppelte Problem des Wärmeaustausches durch Strahlung und freie Konvektion mit turbulenter Grenzschicht in einer volumetrisch beheizten, horizontalen und als grau anzusehenden Flüssigkeitsschicht wird analytisch untersucht. Die Flüssigkeit wird oben durch eine feste isotherme Wand und unten durch eine feste adiabate Wand begrenzt. Es wird eine Näherungsmethode verwendet, die auf der Lösung der Grenzschichtgleichungen beruht, um den Einfluß der wichtigsten Problemparameter auf die Wärmeübertragung an die obere Wand zu erkennen. Für mäßige Prandtl-Zahlen sind dies die Rayleigh-Zahl Ra, die optische Dicke κL und der Kopplungsparameter N zwischen Wärmeleitung und -strahlung. Ebenso wird in dieser Untersuchung das Verhalten der thermischen Grenzschicht an der oberen Wand betrachtet. Bei großem κL ist der Anteil der Wärmestrahlung an der Wärmeübertragung in der Flüssigkeitsschicht für $N > 10$ vernachlässigbar, für $N \sim 1$ mittel und für $N < 0.1$ sehr groß. Jedoch ist bei kleinem κL die Wärmestrahlung nur für $N < 0.01$ von Bedeutung. Da ein höherer Turbulenzgrad eine dünnere Grenzschicht verursacht, ergibt sich ein größerer Einfluß der Strahlung in einer dickeren Grenzschicht. Somit ist bei einer starken Wärmestrahlung eine viel größere Rayleigh-Zahl Ra erforderlich, damit die Lösung der Grenzschichtgleichung gültig bleibt. Unter starken Strahlungsbedingungen zeigt sich, daß selbst bei sehr großen Rayleigh-Zahlen kein Gebiet mit Grenzschichtströmung existiert. Entsprechend wird der Anwendungsbereich der vorliegenden Ergebnisse festgelegt und die Berechtigung der Näherungsmethode begründet. Insbesondere wird die Gültigkeit der vorliegenden Analyse für drei Grenzfälle, $\kappa L \to \infty$, $N \to \infty$ und $Ra \to \infty$ überprüft und ferner durch den Vergleich mit der numerischen Lösung bestätigt.

ЛУЧИСТЫЙ ТЕПЛОПЕРЕНОС В ТУРБУЛЕНТНОМ, ГЕНЕРИРОВАННОМ НАГРЕВОМ, СЛОЕ ЖИДКОСТИ

Аннотация — Проведено аналитическое исследование лучистого переноса и турбулентной естественной конвекции в нагреваемом горизонтальном объеме серой среды, ограниченном сверху жесткой изотермической, а снизу жесткой адиабатической стенками. Для получения зависимости величины теплового потока на верхней стенке от основных параметров задачи, которыми при умеренных значениях числа Прандтля являются число Релея Ra, оптическая толщина κL и параметр взаимодействия теплопроводности с излучением N, использован метод, основанный на приближении пограничного слоя. Найдено, что при большом значении оптической толщины κL доля теплового излучения в общем потоке тепла в слое пренебрежимо мала при $N > 0$, умеренна при $N \sim 1$ и весьма значительна при $N < 0{,}1$. Однако при небольшом значении κL вклад теплового излучения велик только при $N < 0{,}01$. В то время как в пограничном слое меньшей толщины наблюдается более высокий уровень турбулентности, в пограничном слое большей толщины наблюдается более сильное влияние излучения. Таким образом, для того чтобы приближение пограничного слоя оказалось справедливым при наличии интенсивного теплового излучения, требуются гораздо более высокие значения числа Ra. При значительном излучении режим пограничного слоя не наблюдался даже при очень больших числах Релея. В соответствии с этим в работе определены границы применимости полученных результатов и дано обоснование приближенного метода. Справедливость полученных результатов проверена в трех предельных случаях, $\kappa L \to \infty$, $N \to \infty$ и $Ra \to \infty$, путем сравнения с численным решением.